Geometry: 6.1-6.3 Notes

NAME______________________

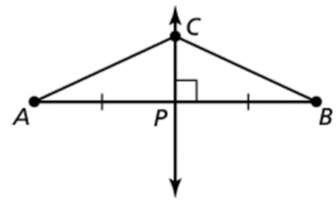
<u>6.1 Use perpendicular and angle bisectors</u>______________________________ <u>Date:</u>________________

Define Vocabulary:

Equidistant

Theorem 6.1 Perpendicular Bisector Theorem

In a plane, if a point lies on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

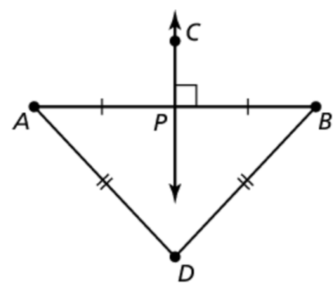


If $\overleftrightarrow{CP}$ is the $\perp$ bisector of $\overline{AB}$, then $CA = CB$.

**<u>Note</u>: A Perpendicular bisector can be a segment, a ray, a line, or a plane.

Theorem 6.2 Converse of the Perpendicular Bisector Theorem

In a plane, if a point is equidistant from the endpoints of a segment, then it lies on the perpendicular bisector of the segment.

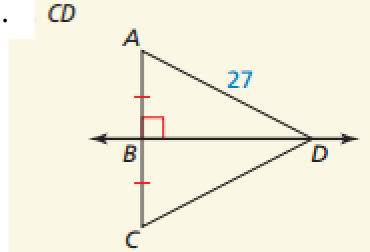


If $DA = DB$, then point D lies on the $\perp$ bisector of $\overline{AB}$.

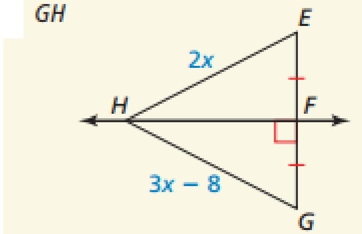
Examples: Find the indicated measure. Explain your reasoning.

<u>WE DO</u>

1. CD

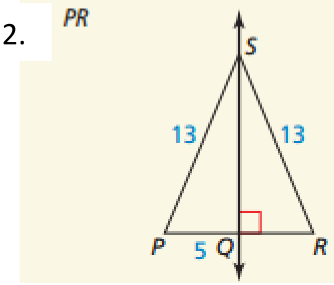


3. GH

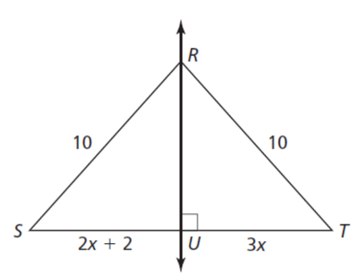


<u>YOU DO</u>

2. PR



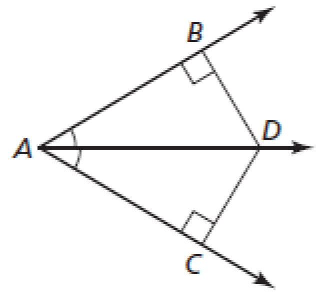
4. SU



Theorem 6.3 Angle Bisector Theorem

If a point lies on the bisector of an angle, then it is equidistant from the two sides of the angle.

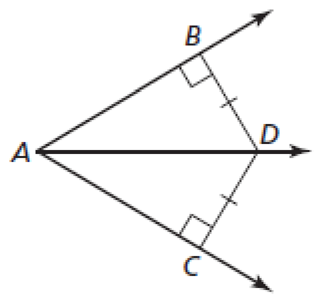
If $\overrightarrow{AD}$ bisects $\angle BAC$ and $\overline{DB} \perp \overrightarrow{AB}$ and $\overline{DC} \perp \overrightarrow{AC}$, then $DB = DC$.



Theorem 6.4 Converse of the Angle Bisector Theorem

If a point is in the interior of an angle and is equidistant from the two sides of the angle, then it lies on the bisector of the angle.

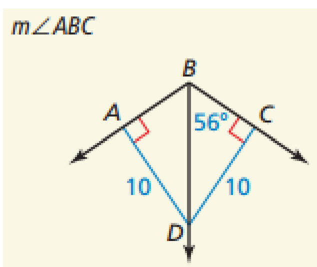
If $\overline{DB} \perp \overrightarrow{AB}$ and $\overline{DC} \perp \overrightarrow{AC}$ and $DB = DC$, then $\overrightarrow{AD}$ bisects $\angle BAC$.



Examples: Find the indicated measure. Explain your reasoning.

WE DO

5.



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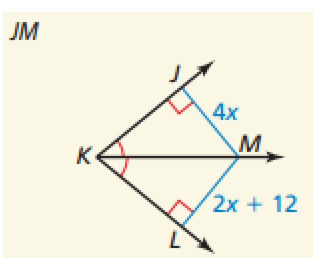
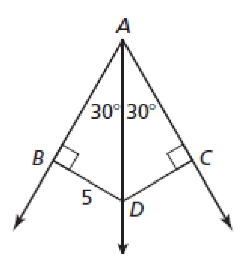
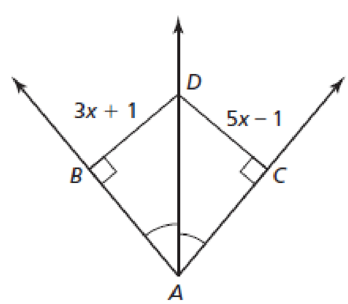


YOU DO

6. DC



8. BD



Steps to find Perpendicular Bisector:

1. Find the midpoint of the segment

2. Find the slope of the segment

3. Then find the perpendicular slope.

4. Using the perpendicular slope and midpoint, find the equation of the perpendicular bisector.

Examples: Write an equation of a perpendicular bisector of the segment with the given endpoints.

WE DO

9. D(5, -1) and E(-11, 3)

YOU DO

10. A(0, -2) and B(2, 2)

Assignment	

6.2 Use bisectors of triangles Date:______________
Define Vocabulary:

concurrent

point of concurrency

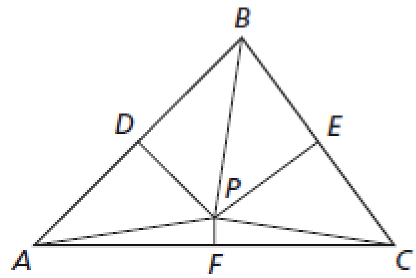
circumcenter

incenter

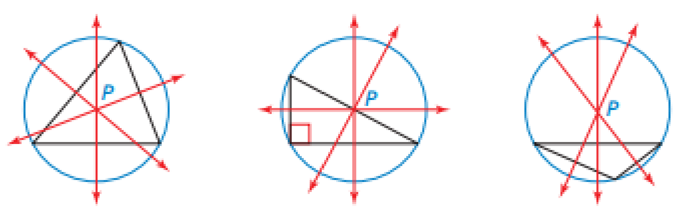
Theorem 6.5 Circumcenter Theorem

The circumcenter of a triangle is equidistant from the vertices of the triangle.

If $\overline{PD}$, $\overline{PE}$, and $\overline{PF}$ are perpendicular bisectors, then $PA = PB = PC$.



The circumcenter P is equidistant from the three vertices, so P is the center of a circle that passes through all three vertices. As shown below, the location of P depends on the type of triangle. The circle with center P is said to be *circumscribed* about the triangle.



Acute triangle
P is inside triangle.

Right triangle
P is on triangle.

Obtuse triangle
P is outside triangle.

Steps to find the Circumcenter:

1. Graph the triangle

2. Find the perpendicular bisectors of 2 sides (horizontal and vertical sides if possible).

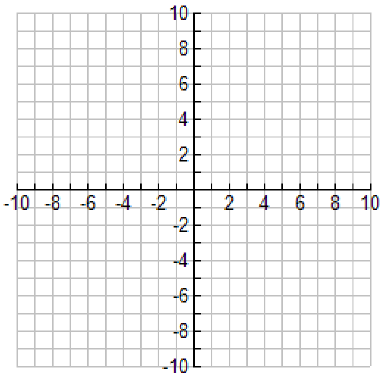
3. Find the midpoint of the remaining side to verify the x-coordinate of the circumcenter.

4. The intersection of the perpendicular bisectors is the circumcenter.

Examples: Find the coordinates of the circumcenter of the triangle with the given vertices.

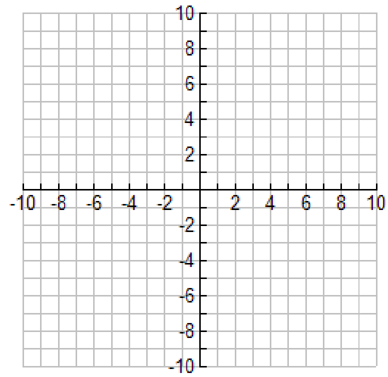
1. **WE DO**

D(6, 4), E(-2, 4), F(-2, -2)



2. **YOU DO**

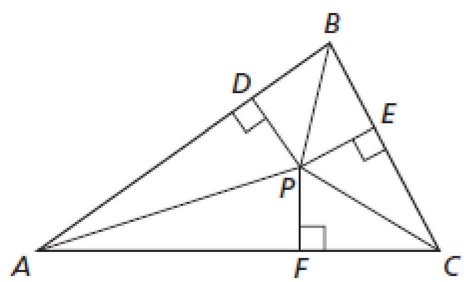
R(0, 0), S(-4, 0), T(-6, 6)



Theorem 6.6 Incenter Theorem

The incenter of a triangle is equidistant from the sides of the triangle.

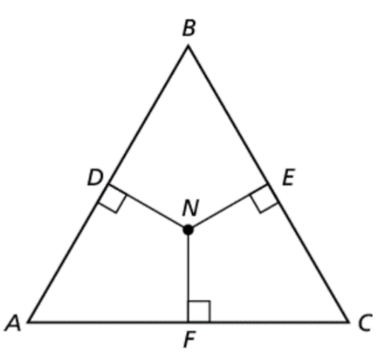
If $\overline{AP}$, $\overline{BP}$, and $\overline{CP}$ are angle bisectors of $\triangle ABC$, then $PD = PE = PF$.



Examples: N is the incenter of the triangle. Use the given information to find the indicated measure.

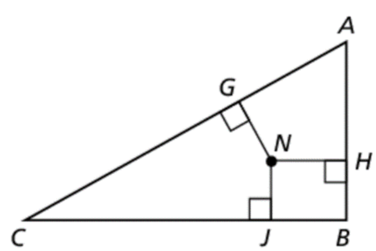
3. **WE DO**

$ND = 2x - 5$
$NE = -2x + 7$
Find NF.



4. **YOU DO**

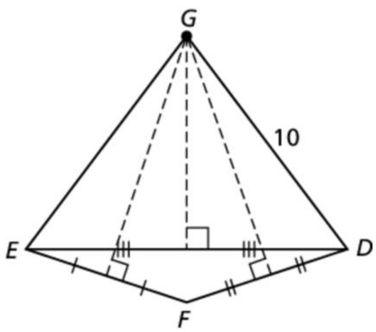
$NG = x - 1$
$NH = 2x - 6$
Find NJ.



Examples: Find the indicated measure.

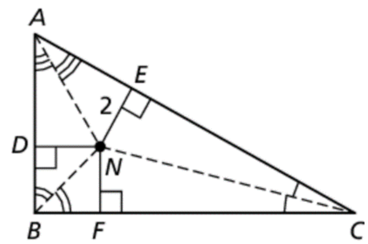
WE DO

5. GE

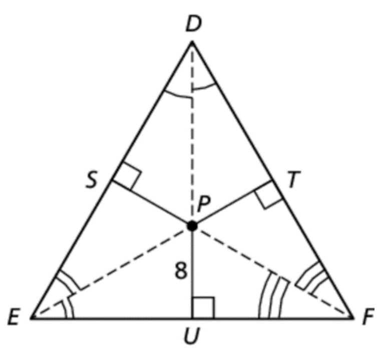


YOU DO

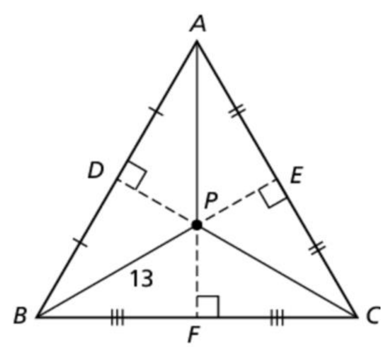
6. NF



7. PS



8. PA



Assignment	

6.3 Using Medians and Altitudes of Triangles.__________________________________Date:______________

Define Vocabulary:

median of a triangle

centroid

altitude of a triangle

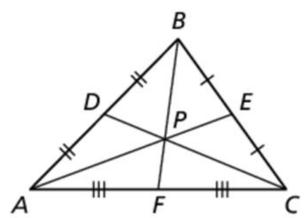
orthocenter

Theorem 6.7 Centroid Theorem

The centroid of a triangle is two-thirds of the distance from each vertex to the midpoint of the opposite side.

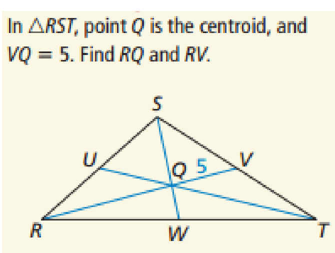
The medians of ΔABC meet at point P, and

$AP = \frac{2}{3}AE$, $BP = \frac{2}{3}BF$, and $CP = \frac{2}{3}CD$.



Examples: Using the Centroid of a triangle.

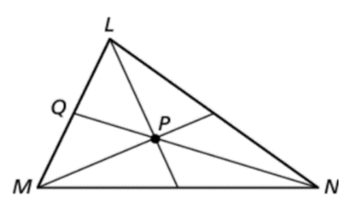
WE DO

1. 

YOU DO

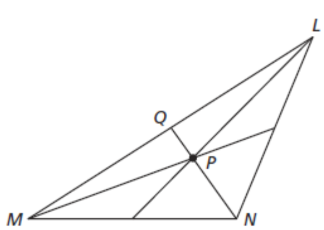
2. Point P is the centroid. Find PN and QP.

$QN = 33$



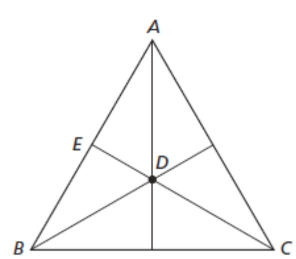
3. Point P is the centroid. Find PN and QP.

$QN = 39$



4. Point D is the centroid. Find *CD* and CE.

$DE = 7$



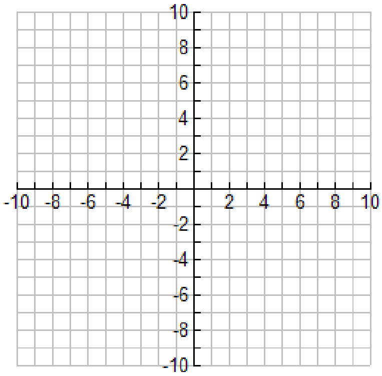
Steps to find the Centroid of the triangle:

1. Graph the triangle
2. Find the midpoint of two of the sides
3. Then connect the midpoint with the opposite vertex of the triangle.
4. Repeat steps 2-3 for the remaining sides
5. Point of intersection is the centroid.

Examples: Find the coordinates of the centroid of the triangle with the given vertices.

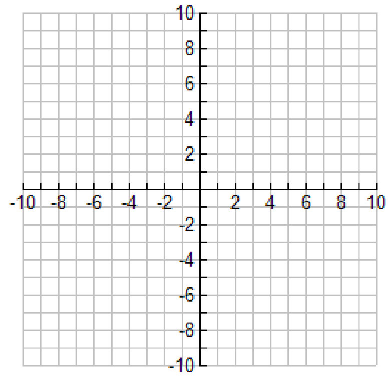
<u>WE DO</u>

5. A(0, 4), B(-4, -2), and C(7, 1)



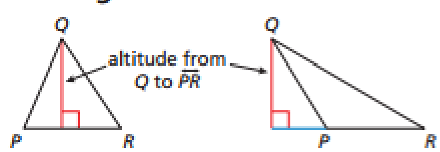
<u>YOU DO</u>

6. F(2, 5), G(4, 9) and H(6, 1)



Using the Altitude of a Triangle

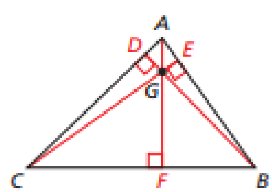
An **altitude of a triangle** is the perpendicular segment from a vertex to the opposite side or to the line that contains the opposite side.



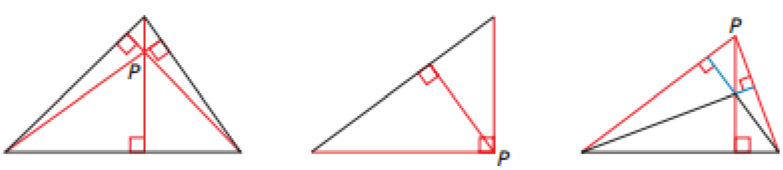
Orthocenter

The lines containing the altitudes of a triangle are concurrent. This point of concurrency is the **orthocenter** of the triangle.

The lines containing $\overline{AF}$, $\overline{BD}$, and $\overline{CE}$ meet at the orthocenter G of $\triangle ABC$.



As shown below, the location of the orthocenter P of a triangle depends on the type of triangle.



Acute triangle
P is inside triangle.

Right triangle
P is on triangle.

Obtuse triangle
P is outside triangle.

Steps to find the Orthocenter of the triangle:

1. Graph the triangle.

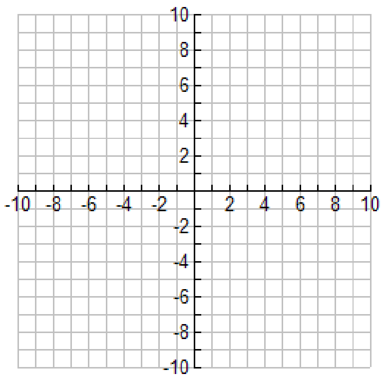
2. Find the equation of the line that contains the altitudes of the 2 sides of the triangle. The line needs to be perpendicular to the sides

3. The point of intersection is the orthocenter.

Examples: Find the coordinates of the orthocenter of the triangle with the given vertices.

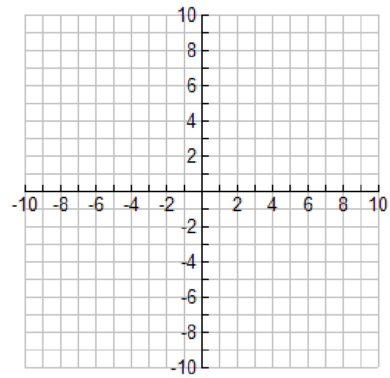
WE DO

7. D(0, 6), E(-4, -2), and F(4, 6)

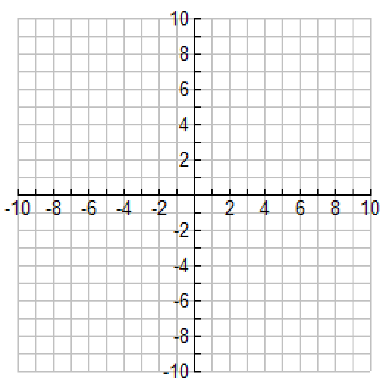


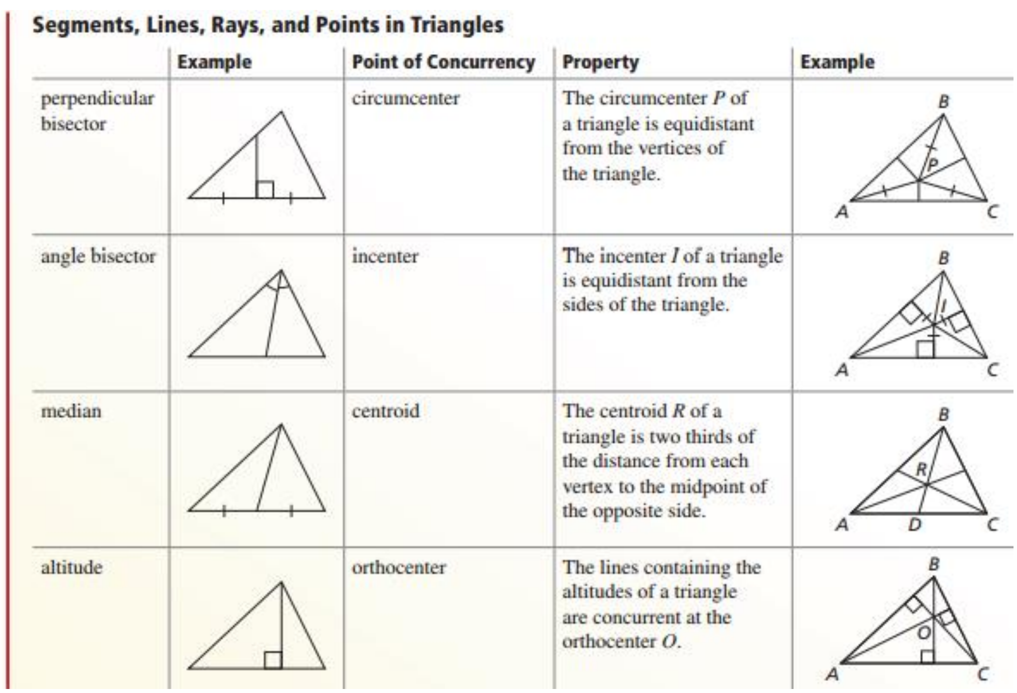
YOU DO

8. J(-3, -4), K(-3, 4), and L(5, 4)



9. D(3, 4), E(11, 4), and F(9, -2)





Segments, Lines, Rays, and Points in Triangles

	Example	Point of Concurrency	Property	Example
perpendicular bisector		circumcenter	The circumcenter *P* of a triangle is equidistant from the vertices of the triangle.	
angle bisector		incenter	The incenter *I* of a triangle is equidistant from the sides of the triangle.	
median		centroid	The centroid *R* of a triangle is two thirds of the distance from each vertex to the midpoint of the opposite side.	
altitude		orthocenter	The lines containing the altitudes of a triangle are concurrent at the orthocenter *O*.	

Assignment	